Linear-Mode Avalanche Photo-Diode Detectors with a Quasi-Deterministic Gain Component: Statistical Model Studies

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ABSTRACT
The optoelectronic gain of a linear mode avalanche photo-diode (APD) results from the cascade of electron and hole impact ionizations that take place in the high-field intrinsic multiplication layer of the APD. Due to the uncertainty associated with the stochastic nature of the APD’s gain, the shot noise present in the resulting photo-generated electrical signal is accentuated and degrades the detection of single photon initiated avalanche signals. Recent advances in linear-mode InGaAs APD detectors have been demonstrated that have reduced excess noise, along with the high gain necessary for detecting single photons. In these devices the avalanche buildup is characterized with a temporally varying noise. At low incident photon / photo-electron levels, the stochastic nature of the impulse response function of these APDs offers the potential of increased probability that the output exceeds a threshold level resulting in a “detection” and, hence, a better receiver-operating-characteristic (ROC). In this paper we examine the ROC (P\text{detection} vs P\text{FalseAlarm}) statistics of these single photon APDs as a function of the quasi-deterministic mean gain and standard deviation for an rms ROIC (readout integrated circuit) noise level of 25e−. Single photo-electron and multiple photo-electron detection statistics are also examined for predicting a ROC. Measured linear-mode APD data are also presented.

Key words: Avalanche photo-diode detectors, linear-mode avalanche photo-diode detectors, direct detection ladar and lidar, receiver operating characteristic (ROC)

1. INTRODUCTION
Some laser radar (ladar) and light detection and ranging (lidar) sensor applications require single photon sensitivity. Furthermore, some applications require that there be no dead-time in the detector's operation, so that downrange target depth measurements can be made in the presence of foreground clutter, bright background sources, or unintended aerosol or surface scatterings which would otherwise cause an APD detector to go blind and stop the detector operation. At near single photon levels, linear mode APD performance is degraded by variations in the amplification gain. This fluctuation in the electrical signal results in an unacceptably low likelihood that the output from the APD will create enough detector amplified signal to exceed a threshold set above the electronic noise floor, generally determined by the read out electronic (ROIC) noise electrons. To minimize excess noise, multiple gain-stage linear-mode APD devices have been demonstrated in recent years which show both high overall gain response as well as a reduced variance on the mean gain value. These APDs also exhibit instantaneous fluctuations of the APD’s impulse response function, where the variance of the initial portion of the impulse response is lower than that of the fully-integrated integrated APD impulse response. In these devices, the initial portion of the avalanche buildup is dominated by a single carrier (electrons) and the electron-initiated ionization chains are characterized by noise lower than the cumulative mean gain, which includes ionization chains that originate from hole feedback in the APD. The temporal dependence of the avalanche process of these devices improves the photon detection efficiency of the devices.

APD noise is most readily characterized by a quantity called the excess noise factor1-4. A mathematical form for this function, the McIntyre formula1-4, has been successfully used to characterize the multiplication noise of conventional thick APDs, where the excess noise factor depends only on the mean gain and on the ratio of the ionization coefficients for holes and electrons. McIntyre’s initial formula rests on the assumptions that the avalanche multiplication region is uniform, the ability of electrons and holes to effect an impact-ionization does not depend on their past history, and that ionization chains are not terminated. However, if any of the above conditions are not satisfied, a more general theory is required for calculating the excess noise factor.
To assess the performance of high-speed APD-based receivers, stochastic models and recursive equations can be used to characterize dead space effects and the strong correlation between the random gain and the duration of the random impulse response so that the mean, variance, and autocorrelation function of the APDs impulse-response function, including the effects of dead space, and hetero-junction multiplication regions, which include the initial energy of carriers arising from heating the carriers prior to their injection into the multiplication region, can be obtained. The effects of individually characterizing the instantaneous noise of the lead edge of the impulse response function and the accumulated mean gain, on the performance of thresholded optical receivers are investigated in this paper. Here, to describe the temporally-dependent performance of the linear mode APD, we simply use two McIntyre functions to describe the impulse response function of the linear mode APD. The excess noise of the initial portion of the impulse response function, which is dominated by single carrier (electron) ionization is described by an effective ratio of ionization rates, characterized by $k=0$, and the integrated mean gain of the impulse response is characterized by $k=0.02$.

In Section 2 the effects of McIntyre electron-gain distributions on the reviewed on the probability of detection at low photon signal levels is characterized. In Section 3 the target and background photon/photo-electron statistics are reviewed as a reference for real system operation. In Section 4 the receiver-operating-characteristic calculation is discussed. In Section 5 the single PE ROC is presented. In Section 6 we repeat this for larger PE count events to see how the statistics improve. We add a quasi-deterministic gain component to determine what is required of this new gain process to achieve a good single photon / photo-electron detection in Section 7. Some system application examples are presented in Section 8 using a useful Monte Carlo code for generating McIntyre electron-gain realizations are described.

2. LINEAR-MODE APD ELECTRON-GAIN FLUCTATIONS

In an APD detector an initial photo-electron (PE) is created when the photon is absorbed, and avalanche multiplication is achieved by impact ionization where an accelerated electron or a hole can collide with a bound valence electron creating an extra hole/electron pair. These additional carriers can gain enough energy from the applied field to cause further impact ionizations until a large pulse of electrons is created. McIntyre has shown that the probability of getting “m” electrons from an avalanche gain region given an input of “q” electrons, is given by:

$$P_{Mc}(m|q) = \frac{q\Gamma\left(\frac{m}{1-k}\right)}{(q(1-k)+k m)(m-q)!} \left(1+k \frac{G_p-1}{G_p}\right)^{\frac{k m}{1-k}} \left(\frac{(1-k)(G_p-1)}{G_p}\right)^{(m-q)}$$  \hspace{1cm} (1)$$

where $G_p = \text{mean PE gain} = \langle m \rangle/q$ ($\geq 1$), and $k$ is the hole to electron ionization rate. Equation (1) agrees with equation (10) in Williams as well as with other recent APD publications.

For single photon/photo-electron studies, the input to the avalanche gain region is set to one:

$$P_{Mc}(m) = P_{Mc}(m|q=1)$$  \hspace{1cm} (2)$$

for getting “m” electrons out following equation (1). In Figure 1 electron count probabilities for a mean gain of $256 (= 2^8)$ and $k = 0.02$ through $0.22$ are shown given one electron input to the gain-region.

Voxtel has developed low noise APD devices with a mean gain characterized by $k = 0.02$ at gain ($G_p$) levels. Other commercial APDs have a $k = 0.16$ to $0.22$, as indicated, and are limited to operating gains of fewer than 25. For any of these APDs, for a single PE input to the electron gain region it is seen that the most likely electron output is one electron. This is obviously not good for the detection of single photon/photo-electron events, and higher gain devices with low variance are needed for good single photon / photo-electron detection.

We note that the McIntyre distribution applies to the total number of carriers generated, without reference to the timing of their generation, and so cannot account for carriers leaving the junction and no longer contributing to photocurrent. What one actually needs for a pulse detection calculation is the distribution of the photocurrent peak pulse-height. That’s where the “quasi-ballistic” component of the gain process enters in as discussed in Section 7 below.
2.1 Extension to multiple serial gain-stage regions showing $k_{\text{effective}}$ approaches zero

Voxtel has developed\[^{10-12}\] multiple gain-stage detectors. We examine this detector statistically at a cursory systems level before proceeding to detection analysis. In this case of assuming single-carrier transport perpendicular to the cross-section, with no carrier feedback from the ionization events between each gain stage, output probabilities from the second gain-stage are then given using the first stage (independent) output probabilities as inputs to the second stage:

\[
P_{M_{2}, 2}(m) = P_{M_{1}, 2}(m | q = 1)P_{M_{1}, 1}(q = 1) + P_{M_{1}, 2}(m | q = 2)P_{M_{1}, 1}(q = 2) + P_{M_{1}, 2}(m | q = 3)P_{M_{1}, 1}(q = 3) + \ldots
\]

where the output electron number “m” becomes the input electron number “q” for the second stage. The output from the third gain stage is then obtained using $P_{M_{2}, 3}(q)$ as the input probability of q electrons as

\[
P_{M_{2}, 3} = \sum_{q=1}^{\infty} P_{M_{2}, 2}(m | q)P_{M_{2}, 1}(q) \quad (P_{M_{2}, 2}(m < q) = 0)
\]

\[
P_{M_{2}, 4} = \sum_{q=1}^{\infty} P_{M_{2}, 3}(m | q)P_{M_{2}, 1}(q)
\]

\[
\vdots
\]

\[
P_{M_{2}, 8} = \sum_{q=1}^{\infty} P_{M_{2}, 7}(m | q)P_{M_{2}, 1}(q)
\]

and so on, as indicated up to stage eight for an eight gain-stage device.
In Figure 2 we plot the electron output distribution for a single stage $G_p = 256$ with $k=0$ as compared to the eight stage, $G_p=2$ each with $k=0.02$, and we see that they are virtually identical. For analysis and for numerical simulations we may approximate the low $k$ eight stage detector with a $k=0$ (single gain-stage) detector with the same net gain as given in equation (1).

Actually, the output from later stages is coupled into the earlier stages by holes\textsuperscript{11-12}. Electrons move from early stages into later stages, but holes carry the opposite charge, and drift in the opposite direction. Consequently, when $k \neq 0$, holes that are generated in later avalanche stages drift back through the structure and constitute some of the input into the early stages. The amplification stages cannot be considered completely independent.

3. PHOTO-ELECTRON STATISTICS REVIEW

For ladar target photons/photo-electrons the probability of “$k$” discrete PE events (which are the input to the APD) occurring in a time interval is given by the discrete negative-binomial\textsuperscript{13-16} distribution probability:

$$P_{NB}^S(k | N_S, M_{spl}) = \Gamma(k + M_{spl}) \left[ 1 + \frac{M_{spl}}{N_S} \right]^{-k} \left[ 1 + \frac{N_S}{M_{spl}} \right]^{-M_{out}} = \int \text{Poisson} \times p_{\Gamma}(I) \, dl$$

This is obtained by averaging the Poisson discrete counting probability density function over the statistics of the aperture-averaged fluctuating received target energy, which is approximately gamma distributed\textsuperscript{13-16} (The probability density function (PDF) is termed the Poisson transform of a gamma distribution. Note that all delta functions in the PDF’s are omitted for clarity.) “$M_{spl}$” is Goodman’s "M parameter" for vacuum propagation. The mean of "$k$" is denoted as $N_S$, where "S" denotes signal, as opposed to dark-counts or background-counts which sum together producing a mean denoted by “$N_n$” corresponding to “noise.” A new $M_{spl}$ occurs every $c \tau_{laser} / 2$ in range when $50\%$ of the surface elements are replaced by new ones, and $M_{spl}$ may be numerically evaluated as described in reference 11. For distant unresolved targets, $M_{spl} \approx 1$ per polarization and per independent laser mode for each $c \tau_{laser} / 2$. If both polarizations are detected $M_{spl} = 2$ per range-bin per laser mode. We also point out that when converting the photon statistics to photo-electron statistics, a new negative-binomial distribution results\textsuperscript{10} having a mean count number which is reduced.
by the quantum efficiency of the photo-cathode surface or semiconductor absorption region ($\eta \times N_S$), yet the $M_{spkl}$ parameter remains the same\(^{11}\).

Background photon produced PE events and dark-count events (those exceeding threshold) are described by Poisson random processes\(^{1,16}\):

$$p_{\text{Pois}}(n \mid N_n) = (N_n)^n \exp(-N_n) / n!$$

When added to the negative-binomial target events, the resulting sum has a PDF which is the convolution of the two individual PDF’s. Goodman\(^{13-16}\) also derives this **signal plus background and dark noise** summation discrete count process as:

$$P_{S,N}(k) = P_S^{NB} \otimes P_{\text{Pois}} = \left( \frac{M_{spkl}}{M_{spkl} + N_S} \right)^{M_{spkl}} \frac{\exp(-N_n)}{\Gamma(M_{spkl})} \sum_{j=0}^{k} \frac{\Gamma(k + M_{spkl} - j)}{j!(k - j)!} \left( \frac{N_S}{M_{spkl} + N_S} \right)^{k-j} (N_n)^j$$

where “$N_n$” is the mean number of background photon plus dark-current-noise counts. It is usually easier just to evaluate the convolution than the summation pdf. When “$N_{\text{sum}}$” pulse returns are added together, $N_S$ becomes $N_{\text{sum}} \times N_S$, $M_{spkl}$ becomes $N_{\text{sum}} \times M_{spkl}$, and $N_n$ becomes $N_{\text{sum}} \times N_n$ in equations (5) through (7).

### 3.1 Gaussian baseline noise and dark count / false alarms for multiple gain-stage APDs

The Gaussian baseline noise for multiple gain-stage state of the art detectors is determined by the readout integrated circuit (ROIC) electron noise plus the dark current from each of the individual detector gain-stages which must be added together\(^9\). If we assume the dark counts originate equally at each of “#stages” high field gain-stage regions (as would be the case for trap assisted tunneling), (1 / #stages) of the dark current sees a gain of 1, (1 / #stages) sees a gain of $G_p$, (1 / #stages) sees a gain of $G_p^2$, etc.

$$G_{dark} = \sum_{n=1}^{#\text{stages}} G_p^{n-1} \quad \therefore e_{dark} / \text{second} = I_{dark,\text{out}} / 1.6\times10^{-19} / G_{dark}$$

The dark electron output pdf is given assuming the dark electron events are not overlapping in time (low dark current) and are independent events immersed in the Gaussian ROIC noise:

$$p_{\text{dark}}(k) = \#\text{stages}^{-1} \sum_{n=1}^{#\text{stages}} p_{M_n}(k \mid G_p^{n-1})$$

where $p_{M_n}$ follows from equation (1) with the appropriate gains from equations (8). The amplified dark electron pdf is convolved with the ROIC Gaussian noise, which in this study is assumed as $1\sigma = 25e^-$.

$$p_n(n) = p_{\text{Gauss}}(m) \otimes p_{\text{dark}}(k)$$

Those dark electrons originating near the input to the APD produce "dark counts" due to their large gain. We assume $10^6$ dark electrons per second (one per $\mu$s). From the noise pdf in equation (10), the probability of a measurement (at $10^6$ per second) exceeding a threshold is determined, and this event is termed a dark count or dark event, as discussed in the receiver operating characteristic (ROC) section below.

### 4. RECEIVER OPERATING CHARACTERISTIC

A “detection” begins when a photon is annihilated and a photo-electron (PE) is created. The probability of a PE creation is proportional to the magnitude squared of the E-field (irradiance). The detector output may be accurately modeled, therefore, as set of probabilities of discrete electron events $>> 0$. The APD amplified PE initiated electron-pulse must then exceed a threshold level, which is used to limit the false alarms produced by baseline noise given in equation (10) from the electronics and detector leakage currents. In the following, **lowercase letters** are the discrete (or continuous) random variable PDFs whereas the **uppercase letters** are the discreet event probabilities which are then multiplied by their corresponding delta functions and summed to form a PDF.

Ordinarily the signal, “s”, plus baseline noise, “n”, output, “r”, would be modeled as:
and the resulting PDF of “r” is given by the convolution of the individual PDF’s:

\[ p_s(r) = p_s(s) \otimes p_s(n) = \rho_{APD}(s) \otimes p_s(n) \]  \hspace{1cm} (11b)

where \( p_{APD}(s) \) is the APD output pdf and \( p_{N} \) is expressed in (discrete) electrons (“e”) or discretized continuous parameters in the computer. The APD electron output PDF is based on the McIntyre electron distribution denoted by “PMc” as described above in equation (1):

\[ p_{APD}(q) = \sum_{s=1}^{\infty} P_{Mc}(m | s) P_s(s) \delta(q - m) = P_{APD}(m) \delta(q - m) \quad (s \geq 1, m \geq s) \] \hspace{1cm} (11c)

\[ P_{APD}(m) = \sum_{s=1}^{\infty} P_{Mc}(m | s) P_s(s) \quad (s \geq 1, m \geq s) \] \hspace{1cm} (11d)

The APD output McIntyre distribution is summed over the (mutually exclusive) signal PE probabilities, \( P_s(s) \). The signal photons/PE’s statistics are given by a negative-binomial distribution discussed above in equation (5) for diffuse targets and equation(7) for target and background photons.

However, for photon processes and the PE counting detectors of interest, when zero photons (and therefore zero PEs) occur in a return measurement, with a finite probability of \( P_s(s=0) \), a sample of the baseline noise “n” is made. This event is mutually exclusive from when one or more PE’s and the resulting avalanche electrons are created, and \( p_{N} \) must be added with weight \( P_s(s=0) \) to the PDF of the sum. The output of the APD detector when there are PE’s is convolved with \( p_{N} \) and added with probability \( (1-P_s(s=0)) \) so that equation (11b) becomes:

\[ p_{S}(r | s, n) = P_{S}(s=0) p_{N}(n) + (1 - P_{S}(s=0)) p_{APD}(s) \otimes p_{N}(n) \]
\[ = P_{S}(s=0) p_{N}(n) + \sum_{s=1}^{\infty} P_{S}(s) P_{Mc}(m | s) \otimes p_{N}(n) \]
\[ r, s, n \in [0,1,2,3,\cdots,\infty) \quad \text{and} \quad m \in [1,2,3,\cdots,\infty), \quad m \geq s \] \hspace{1cm} (11e)

(Note that by adding \( P_{S}(s=0) \times \delta(0) \) to \( p_{APD} \), equation (11b) can be used for notational simplicity.)

4.1 Single photon / photo-electron response simplification

For detector testing purposes in this study we may assume there is always 1 PE produced by the light source and \( P_s(s=1) = 1 \) and therefore \( p_{S}(s) = \delta(1) \), and \( P(s=0) = 0 \) so the first term disappears.

4.2 Receiver operating characteristic

The probability of a false alarm is the probability that a Gaussian baseline noise event with no signal present exceeds a threshold:

\[ P_{FA}(n_{th}) = \sum_{n=n_{th}}^{\infty} p_{S}(n) \] \hspace{1cm} (12a)

denoted as “H0” for hypothesis “0” (resulting in an electron noise count), and the probability of detection is given by

\[ P_{D}(n_{th}) = \sum_{r=n_{th}}^{\infty} p_{S}(r) \] \hspace{1cm} (12b)

summing above the same discrete threshold, given that a signal is present, denoted “H1”. Equations (12a and 12b) constitute a “receiver-operating-characteristic” or ROC.

5. SINGLE PHOTON / PHOTO-ELECTRON R.O.C.

A single dark electron for each gain stage is input into the McIntyre gain equation and a false alarm probability distribution is computed assuming a ROIC Gaussian noise of \( \sigma = 25e^{-} \) following equation (10) for various number of stages and various gain per stage. A typical \( P_{FA} \) vs threshold is shown in the left side of Figure 3. A probability of 0.01 and 0.002 are indicated and assuming 1x10^6 dark electrons per second are created throughout the device, this would
result in 10,000 or 2,000 dark events or dark counts per second. A single signal photo-electron is then used in the McIntyre distribution for the entire gain of the device, $G_{p\text{stages}}$, and convolved with the noise distribution following equation (11b) or (11e). The probability of detection is illustrated in the right side of Figure 3 and the corresponding value for the two thresholds for 10,000 DCR and 2,000 DCR are indicated. Table I summarizes the results for gain stages of 10 through 14 and stage gains of 2.0 through 2.4 each. Table II summarizes the results for .002 $P_{FA}$ or 2,000 dark events per second.

![Figure 3](image.png)

Figure 3. Probability of a false alarm versus threshold level for a 14 stage device with gain per stage of 2.1, left side. Right side shows the detection probability given a single photo-electron at the input of the detector. At $10^6$ dark $e^-$ per second, the false alarm rate is, therefore, 10,000 or 2,000 events per second.

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Table I. Detection Probability @ $P_{FA} = 0.01$ or $\sim 10,000$ Dark Events per Second, One PE Input

Table II. Detection Probability @ $P_{FA} = 0.002$ or $\sim 2,000$ Dark Events per Second, One PE Input
For a system applications ROC calculations (as in equation (11)), if the background photons/photo-electrons equal or exceed 10,000 per second, then the lower threshold setting could be used, thereby obtaining the higher detection probabilities.

6. LARGER PHOTO-ELECTRON INPUT EFFECTS ON PERFORMANCE

When more than one photo-electron is created at the input to the gain regions, the electron output statistics improve greatly. In this section we calculate the ROC for two, three, and four photo-electrons created by the target within a reciprocal detector bandwidth as described by the McIntyre distribution of equation (1). An example is shown in Figure 4 for a 12 stage device with G_p = 2.1 per stage and two photo-electrons created within a detector reciprocal bandwidth.

In Table III the detection probability is summarized for the two threshold settings of .01 (~10,000/s DCR) and .002 (~2,000/s DCR) versus the number of gain stages and the gain per stage, G_p. We observe that 3 PEs are required to achieve a P_D > 0.9 at the higher dark count rate. At the lower dark count rate about 4 PEs are required to achieve the same P_D. Typical single gain-stage APDs usually require around 25 PEs for an 0.9 P_D at P_FA = 10^{-3}.
Table III. Detection Probability versus Number of Photo-Electrons

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7. EFFECT OF A QUASI-DETERMINISTIC GAIN COMPONENT

Recent measurements of the single carrier dominated multi-stage APDs at Voxtel have demonstrated behavior indicative of temporally dependent instantaneous noise during the avalanche buildup time caused by photon events. This behavior is a result of temporal separation of the initial electron-initiated avalanche response of the APD, and the avalanche processes that result from the ionization chains resulting from the hole feedback in the device. This results in an instantaneous variance of the mean gain throughout the impulse response of the APD. The signal from the early portion of the impulse response, where threshold detection is performed, is stable from electron pulse to electron pulse. Hence, we call the gain nearly fixed or "quasi-deterministic."

In this section we add a fixed electron number to the electron output as a fraction of the total mean McIntyre gain to scope the effects. In Figure 5 a 12 stage device with gain per stage of 2.1 and with a 10% additional quasi-deterministic gain for each of two PEs is shown. This is to be compared to Figure 4 above which is without the additional fixed gain. It can be seen that P0 increases from 0.74 to 0.81 at ~10,000/s DCR and from 0.49 to 0.56 at ~2,000/s DCR with the 10% of McIntyre gain added as a fixed gain.

In Table IV quasi-deterministic gains of 10%, 20% and 30% of the McIntyre gain are added to the McIntyre "tail" electron outputs to determine the magnitude of this effect. We see that the quasi-deterministic gain has a small effect at

Table IV. Detection Probability versus Additional Quasi-Deterministic Gain at 1PE Input

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<th>1PE</th>
<th>+10% G</th>
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<th>+30% G</th>
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<th>+10% G</th>
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Figure 5. Same as Figure 4 but with 10% quasi-deterministic gain added with 2 PEs input example.

these percentages of the total McIntyre gain. This physical phenomena is new and is under investigation at Voxtel.

The avalanche process inside the APD is not instantaneous, but the current through the APD is proportional to the instantaneous carrier count in the junction. That means the carriers generated by avalanche appear over a certain span of time, and the peak response of the APD is proportional to the maximum number of carriers instantaneously present in the junction, rather than the total number of carriers generated. If the avalanche process takes a long time to complete (which is true for high average gain), then some of the carriers created early on in the process will have already left the junction before the last carriers are generated. The McIntyre distribution applies to the total number of carriers generated, without reference to the timing of their generation, and so cannot account for carriers leaving the junction and no longer contributing to photocurrent. This additional effect makes the k=0 distribution more appropriate. What one actually needs for a pulse detection calculation is the distribution of the photocurrent peak pulse height. This is where the “quasi-ballistic” component of the gain process helps. Improved detector performance is anticipated with refined gain structure design.

8. MONTE CARLO REALIZATION COMPARISON

Generating Monte Carlo realizations of the McIntyre electron gain distribution of equation (1) is not straightforward. Fortunately the MATLAB "slicesample" function can be set to generate accurate random numbers for arbitrary function shapes. Random dark electron output numbers were generated from each of 11 gain stages, each stage having a gain of 2 following equation (9). These outputs were added to zero mean Gaussian $\sigma = 25e$ electrons individually, once every
microsecond over a 10 μs period in an 0.5 μs bin. This total baseline + dark e noise was then randomized in time. At
the center of each 10 μs period, a single electron amplified output of $2^{11} = 2048$ gain realization was added to the random
Gaussian floor. One hundred realizations are shown in Figure 6. At a threshold setting of 2,500e corresponding to one
false alarm per 100 or .01 rate, we see that 35 detections would have been made. This is in good agreement with the
detection probabilities listed in Table I for single PE detection at 11 stages of gain = 2 each. This is a first order Monte
Carlo simulation and is used for illustration of the concepts. More detailed modeling is under investigation.

![100 Monte Carlo realizations for an 11 gain-stage device with Gp = 2 each stage and a single PE input and one dark
electron per microsecond. A ROIC 1 σ of 25e is also assumed. Above a threshold of 2,500 for a P_{FA} of ~.01, we observe 35
detections in good agreement with Table I for #stages = 11 and Gp = 2.](image_url)

9. SUMMARY AND CONCLUSIONS

Newly developed multiple gain-stage InAlGaAs linear-mode APDs are under study for single photon / photo-electron
detection performance, particularly in the near infra-red region of interest to ladar and lidar researchers. An accurate
detector dark current and dark event model PDF is presented based on recent experimental data. This dark electron PDF
is convolved with a zero mean Gaussian ROIC noise of $1\sigma = 25e^-$ to produce the noise PDF. Under the McIntyre
approximation single photon / PE events are shown to be detected at probabilities of 0.34 to 0.58 at a dark count rate of
about 10,000 /s. At a lower 2,000 /s DCR, the probabilities of single PE detection drop to 0.16 to 0.31. Detection
probabilities of >0.9 require 3 PEs at the high DCR and require about 4 PEs at the lower DCR. (The quasi-deterministic
gain contribution was not included in the multiple PE cases of Table III, only in the associated figure for illustration.)
This represents almost an order of magnitude improvement over previous single stage APD detectors.

Currently the detector multiple gain-stage structure is being optimized for minimizing dark electrons and creating lower
noise electron amplification. The "quasi-ballistic" electron gain effect is under study, and it is hoped that this
phenomenon can be used to improve detector performance in the future. Analytical and numerical equations are now
being developed to determine the electron current peak pulse height distribution (PHD) for detection enhancement, as
opposed to the McIntyre total electron output equation. The results of these studies and the improvements to photo-
electron detection will be reported in future papers.
10. ACKNOWLEDGEMENTS

This study was performed under continuing SPARTA Inc. dba Cobham Analytic Solutions and under Voxtel, Inc. internal research and development efforts to apply advances in signal processing and algorithm research to laser radar systems and object feature estimation.

REFERENCES